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Wavelets for Compression of Image-like Data

FINAL REPORT

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Table of Contents

I. List of manuscripts prepared under SDIO/IST-ARO sponsorship

Books

(1) C. K. Chui and G. R. Chen, *Discrete H^∞ Optimization*, Springer-Verlag, 1997.

(2) C. K. Chui, *Wavelets: A Mathematical Tool for Signal Analysis*, SIAM Publ., Philadelphia, 1997.

Research Articles

(1) C. K. Chui, D. Hong, and R. Q. Jia, Stability of optimal order approximation by bivariate splines over arbitrary triangulations, *Trans. Amer. Math. Soc.* **347** (1995), 3301–3318.

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(10) C. K. Chui and X. L. Shi, A Study of biorthogonal sinusoidal wavelets, in *Curves and Surfaces with Applications in CAGD*, A. Le Méhauté, C. Rabut, and L. L. Schumaker (eds.), Vanderbilt University Press, Nashville, 1997, pp. 51–66.

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(23) C. K. Chui and X. L. Shi, Spectral characterization of dual affine frames with arbitrary scales, under preparation.

(24) C. K. Chui and W. J. He, Multiresolution tight frames, under preparation.

(25) C. K. Chui and X. L. Shi, Dual affine frames governed by phase shift-invariance, under preparation.

II. Participating Scientific Personnel

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III. Brief Outline of Research Findings

The objective of this report is to give a brief descriptive summary of the research findings on the research project under ARO Contrat #DAAH04-95-1-0193, sponsored by SDIO/IST and managed by the U.S. Army Research Office. This is a three-year research grant with a one-year no-cost extension. Technical details are not included in this report, since most of the results have been reported semi-annually.

The research findings can be divided into five groups. Group 1, which consists of papers 19, 20, and 21, is aimed at building the mathematical theory for our research on formulation and characterization of wavelets, splines, and other local basis functions. Group 2, which consists of papers 2, 3, 15, 16, 23, 24, and 25, is a compilation of research work on wavelet foundation, theory, methods, algorithms, and computational schemes. Group 3, which consists of papers 1, 4, and 9, is focused on the stability study and approximation order of bi-variate splines. Group 4, which consists of papers 5, 10, 11, 12, 17, 18, and 22, is focused on the third type of local basis functions, namely, localized sinusoidal functions. Finally, Group 5, which consists of papers 6, 7, 8, 13, and 14, is concerned with applications, particularly on wavelet feature extraction and compression of image-like data.

1. Mathematical Theory [19, 20, 21]

For the purpose of constructing useful basis functions that meet certain specific needs, it is best to first build a powerful mathematical structure that characterizes such basis functions and that can be used fairly easily for identifying such basis functions. The best example is the notion of multiresolution analysis that has its origin from computer vision and was introduced by S. Mallat and Y. Meyer to the mathematics community for constructing (orthonormal) wavelets. In fact, the famous compactly supported orthonormal wavelets of I. Daubechies were constructed by using the scheme of multiresolution analysis. However, this mathematical scheme has a few shortcomings that prevent its applications to constructing basis functions to need certain requirements. For example, when we use an irrational number (greater than 1) as the scale (or dilation) constant, then an orthonormal basis that corresponds to some multiresolution analysis does not even exist, although it can be shown by using the framework developed in our work [19, 20] that an orthonormal wavelet basis with an arbitrary irrational dilation constant greater than 1 can be constructed. Another example is that when the multiresolution analysis structure of a B-spline (of order greater than 1) is used to construct compactly supported tight frames of splines, we need at least two frame generators, although there are examples where one generator is sufficient. In our work [19], we develop a new mathematical structure based on shift-invariance; and in [20], we develop a somewhat equivalent mathematical structure based on commutation with the shift operations. The work in [19] is restricted to the Hilbert space setting, while the approach in [20] is more general. Both approaches are designed to characterize biorthogonal bases (particularly orthonormal bases) and dual frames (particularly tight frames). Although these approaches, including the structure of multiresolution analysis, can be adopted for the periodic setting, they do not facilitate in adding new features. In [21], we consider the interpolation scheme for the periodic setting

and prove that it is closely related to the Marcinkiewicz-Zygmund inequalities and A_p weights in harmonic analysis. The study in [21] is still preliminary. Much more work is required to understand the significance of this approach for constructing periodic wavelets and localized sinusoidal bases.

2. Wavelets and Affine Frames [2, 3, 15, 16, 23, 24, 25]

One of the features that are frequently required in applications is that the scaling function from which the wavelets are generated has the Legrange interpolating property. These wavelets are called interpolating wavelets in the wavelet literature. In [2], we give a complete characterization of interpolating wavelets, classify these wavelets into two types, formulate the decomposition and reconstruction algorithms, and introduce the notion of functional wavelet transform with dual distributions as analyzing wavelets.

In some applications, it is desirable to use spline functions to construct interpolating wavelets. In [3], we apply the "blending algorithm," using the most local (but otherwise useless) spline interpolants as building blocks for the blending scheme to construct interpolating spline wavelets. A very general scheme for constructing spline wavelets with arbitrary knots and restricted to a bounded interval is introduced in our work [15]. This paper is devoted to developing very efficient computational algorithms, that include construction of the wavelets and realization of the orthogonal wavelet decomposition and reconstruction, without leaving the spline domain. The compactly supported spline wavelets in [3, 15] have no hope to be able to perform both analysis and synthesis with finite time duration without a change of bases, unless the basis structure is not required. The approach in our work [24] under preparation is to use compactly supported tight frames of spline functions. This topic of study was initiated and popularized by A. Ron and X. Shen, who build tight frames by using m generators, where m is the order of the splines. So, for linear splines, they use 2 generators and for cubic splines, they use 4 generators, etc. In our paper [24], we construct compactly supported spline tight frames by using exactly 2 generators, for all order m. To do this, we discover a necessary and sufficient condition for the existence of multiresolution tight frames, and observe that all B-splines, regardless of the order, satisfy this condition. However, in contrast to the belief of Ron and Shen, multiresolution tight frames do not always exist, since the condition we discover is not satisfied by a certain class of scaling (or refinable) functions. On the other hand, we do believe that tight frames always exist but they may not come from a multiresolution analysis. The reason for this conjecture is that in [25] we have developed a characterization of dual frames (and particularly tight frames) by using phase-shift invariance. It is somewhat surprising that spatial-shift invariance is a consequence of phase-shift invariance for the affine setting. So the general theory in [19] is easier to use for affine wavelets.

If quasi-affine frames are considered, spatial-shift invariance is free, and this is a topic of discusssion in [16]. However, the condition of phase-shift invariance still has to be satisfied for duality. All the papers discussed above are concerned with affine wavelets and wavelet frames with dilation (or scaling) constant equal to 2. In [23], we consider the general setting of arbitrary dilation constants different from 1. This paper, under preparation, gives necessary and sufficient conditions on the Fourier transform of frame (or wavelet) generators for biorthogonality and duality. As an example, we are able to

construct orthonormal wavelets with irrational dilation that give reasonable time-frequency localization. This answers a question of existence of such wavelets by I. Daubechies.

3. Bivariate Splines [1, 4, 9]

When two-dimensional data sets are considered, the most common approach is to use tensor-products of one-dimensional basis functions. This usually works well when the data sample points are placed on rectangular grids. However, to generalize univariate splines and wavelets on non-uniform knots, as studied in [15], to the bivariate setting and to do a better job than using the tensor-product B-splines and B-wavelets, one must consider arbitrary triangulations. It is a well-known fact due to C. de Boor and K. Höllig, however, that to construct a local basis of bivariate splines on an arbitrary triangulation with r^{th} order of smoothness, the total polynomial degree must be at least $3r+2$. In an earlier work by M. Lai and the P.I., such local bases were constructed, but the local basis functions are not stable in the sense that the constant of best-order approximation depends on the near-singularity of the triangulation. In [1], we give a stable construction by allowing the supports to be slightly larger. For $r = 1$, to reduce the polynomial degree from $3\times1+2 = 5$ to 4, it is possible to lose in the order of approximation. In [9], we study this topic of order of approximation and develop a mathematical procedure to construct a local basis to achieve this optimal order of approximation. In paper [5], we show that it is sometimes possible to increase the optimal order of approximation by 1, by strategically swapping certain edges of the triangulation. This is the first paper in the literature that suggests this "approximation-order-increase" strategy. Observe that swapping edges does not increase the number of triangles and does not introduce new vertices. It is interesting to point out that even the triangulation by the three-directional mesh of the uniform rectangular grid requires a small percentage of edge swapping to increase the approximation order.

4. Localized Sinusoids [5, 10, 11, 12, 17, 18, 22]

In preparation for our study of localized sinusoidal basis functions with arbitrary partitions and arbitrary window functins, we first considered sinusoidal functions on non-uniform grid partitions. In other words, we dealt with non-periodic and non-orthogonal sinusoids, which do not constitute an unconditional basis in general. So, the problem is how much the uniform grid can be perturbed so that the non-periodic trigonometric system remains to be complete and stable (i.e. a frame). For the univariate setting, the maximum perturbation tolerance is $\frac{1}{4}$, and this is called the Kadec $\frac{1}{4}$-theorem. In [5], we generalize the Kadec $\frac{1}{4}$-theorem to the multivariate setting. In [10], we considered two most important classes of localized sinusoidal basis functions, usually called "cosine wavelets." The first class is to allow an arbitrary partition of the real line and window functions of arbitrary shapes but small supports. The supports are so small that only adjacent windows are allowed to overlap. Of course, this class includes the original work of Malvar and of Coifman and Meyer, as well as the various generalizations by Auscher, Weiss, Auscher, and others. For this reason, we adopt the original name of "localized cosines" for this class of cosine wavelets. The second class of cosine wavelets is restricted to uniform partition of the real line, but allowing the window functions to be arbitrary. More specifically, the window

functions of different intervals of the partition may be completely different, and they are allowed to have infinite support. Since this class was first studied by K. Wilson et al., we called this class of cosine wavelets by the name of "Wilson wavelets." The paper [10] is an anouncement and detailed descriptions of our approaches and preliminary results on both localized cosines and Wilson wavelets. The paper [12] is the full paper on localized cosines. This paper unifies all the previous work and includes a complete characterization of dual localized cosine bases, as well as their explicit formulation. The main results also inlcude a theorem on the equivalence of dual Riesz basis, dual frames, and formulation of the sharp frame and dual frame bounds. The paper [11] is a generalization of the result on complete characterization of dual windows in [12] to the multivariate setting. Several examples for practical applications are also given in [11]. In applications, it is important to be able to change a sinusoidal formulation to a localized cosine representation. In [17], we derive all the window functions that allow such transition. In other words, the main results in [17] are formulations of window functions for localized cosines that ensure that all sinusoidal functions are preserved. Marsden-type identities are also given in [17]. In [18], we give the complete results on Wilson wavelets that are parallel to those for localized cosines in [12]. These results unify all the previous work by Wilson et al. Coifman and Meyer on Gabor bases; Daubechies, Jaffard, and Journé on characterization in the Fourier domain; as well as Jawerth and Sweldens on more restrictive dual windows. A computational procedure for finding all dual window functions is also given in [18]. Finally, in our work [22], a complete characterization of all cosine wavelets (including both localized cosines and Wilson wavelets) is given in terms of shift-invariance. The multivariate setting is considered in [22], and it is somewhat surprising to find out that for cosine wavelets, phase-shift invariance is a consequence of spatial-shift invariance. Hence, the general theory in [19] is much more refined for this special class of basis functions. Recall from [25] that for affine wavelets and frames, spatial-shift invariance is a consequence of phase-shift invariance instead.

5. Applications [6, 7, 8, 13, 14]

Two specific areas of applications of wavelets to medical image analysis were investigated: medical image compression [6, 8] and medical image feature extraction [7]. In both of these applications, the medical image is first mapped to the spline/wavelet domain. In other words, instead of representing the medical image by a tensor-product spline series, we only use a spline series with a very coarse knot sequence to represent the blur (i.e. D.C.) component of the image, while we keep the residue of the image in the wavelet domain in terms of tensor-product B-spline and B-wavelet series. The coefficients of the residue series representation are discrete wavelet transform (DWT) of the image, unfortunately using the dual (i.e. biorthogonal) B-spline and B-wavelet as analysis (or integral) kernels. These duals are not as good as the original B-spline and B-wavelet in lowpass and bandpass filtering, in the sense that they provide larger time-frequency localization windows and larger side-lobe to main-lobe ratios in their spectra. For this reason, we apply the duality principle introduced by the P.I. in 1991 to apply the B-spline and B-wavelet both for image representation and for image lowpass/bandpass image analysis. Consequently, very compact histograms of the residue image are obtained. For image compression, a scalar

quantization scheme greatly reduces the image redundancy due to the compactness of the histograms. For feature extraction, the high-frequency features stand out more clearly again due to the compactness of the histogram that facilitates separation of high-frequency features from other high-frequency image details.

To take care of random noise while ensuring optimal estimation, a Kalman-filter-like decomposition algorithm is developed in [13]. The optimality of this algorithm is based on linear unbiased minimum-error variance least-squares estimation. The efficiency of this wavelet decomposition algorithm is achieved in a recursive manner. In [14], this approach is further refined and extended to simultaneously perform estimation of unknown random signals and decomposition of the estimated signals into wavelet sub-bands in real-time. A set of Monte Carlo simulations is also discussed in [4], and statistical performacne tests showed that this simultaneous estimation and decomposition approach even outperforms standard Kalman filtering. Further study in extending this to two-dimensional data sets is needed to efficiently process image-like data.